2022-26 PHYSICS (IRC)
Time:3 hours
Full Marks: 75
Answer the questions as per instruction given
The figures in the right hand margin indicate marks
Candidates are required to give answers in their own words as far as practicable.

Group-A
(Short Answer Type Questions)
Answer all the following questions.

1. Answer the following questions in a few words or maximum in one sentence. (5×1=5)
   (a) Write the scalar triple product of $\vec{A}, \vec{B}$ and $\vec{C}$ in terms of rectangular components.
   (b) Write the differential equation of motion for a particle executes SHM.
   (c) Write the relation between the elastic constants , η and K.
   (d) What are the unit and dimensions of surface tension?
   (e) What is the second fundamental postulate in postulates of special theory of relativity?
2. What do you mean by the mean free path of the molecules of a gas? (5)
3. State and explain the zeroth law of thermodynamics. Explain the concept of (5) Temperature on the basis of zeroth law.

Group-B
(Long/Descriptive Answer Type Question)
Answer any four of the following questions.

4. State and Prove Gauss's divergence theorem. (5+10)
5. (a) Find an expression for the velocity of a simple harmonic oscillator. (7)
   (b) Velocity of simple harmonic oscillator at any time t leads the displacement (8)
   By a phase angle $\frac{\pi}{2}$ radian. Explain why?
6. What is angle of twist? Deduce an expression for the couple required to twist (5+10)
   A uniform solid cylinder by an angle.
7. Define coefficient of viscosity of a liquid. Derive Poiseuille's formula for capillary
   flow of liquid. (5+10)
8. Write down Lorentz transformation equations and hence explain Length (5+5+5)
   Contraction and Time dilation.
9. What are transport phenomena in gases? Obtain on the basis of kinetic theory (5+10)
   An expression for the coefficient of viscosity of a gas and show that it is independent
   Of pressure and varies directly as square root of absolute temperature.
10. Apply the first law of thermodynamics to discuss the following processes. (5×3=15)
    (i) A cycle of processes,
    (ii) Isobaric process,
    (iii) Isochoric process,
    (iv) Adiabatic process and
    (v) Free expansion

Answers
Group-A

1. (a) $\vec{A}.(\vec{B} \times \vec{C}) = A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y - B_yC_x)$

(b) $\frac{d^2y}{dt^2} + \omega^2 y = 0$

(c) $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$

(d) Newton per meter and $[M^1L^0T^{-2}]$

(e) The velocity of light in vacuum is independent of the relative motion of the source or the observer.

2. The mean free path is the average distance travelled by a molecule between two successive collisions with other molecules.

According to the kinetic theory, the molecules of a gas are constantly moving In all directions and with various speeds. They frequently collide with one another when their Speeds and directions change. They do not exert any force upon one another except at Collision. Therefore they move in straight lines with constant speeds between two successive collisions. Hence if we watch a particular molecule it will be found to have short zig-zag paths of different lengths. These are called the free paths of the molecule and their mean is called the mean free path.

3. Zeroth law is the fundamental law in thermodynamics. It states that if two systems A and B are separately in thermal equilibrium with a third system C ,then A and B are in thermal equilibrium with each other.

Suppose there are three systems A, B, and C . System A and B are isolated from each Other but are in thermal contact with C. Experiments show that both A and B individually attain thermal equilibrium with C. If now A and B are put in thermal contact of each other, no further change takes place. That is A and B are found to be in thermal equilibrium with each other.

The zeroth law can be used to define temperature. It leads to that all systems in thermal equilibrium with one another have a common property having same value for all of them. This property is temperature. Thus , the temperature of a system is the property which determines whether or not the system is in thermal equilibrium with other system. That is , two systems in thermal equilibrium with each other are at the same temperature. Conversely , if two systems have different temperatures, they cannot be in thermal equilibrium with each other.

4. Statement:-"According to this theorem the surface integral of a vector field $\vec{A}$ over a closed surface $'s'$ is equal to the volume integral of the divergence of a vector field $\vec{A}$ over the volume V enclosed by the surface"

$$\oiint_S \vec{A}.d\vec{s} = \iiint_V (\vec{\nabla}.\vec{A})\, dV$$

Proof :-
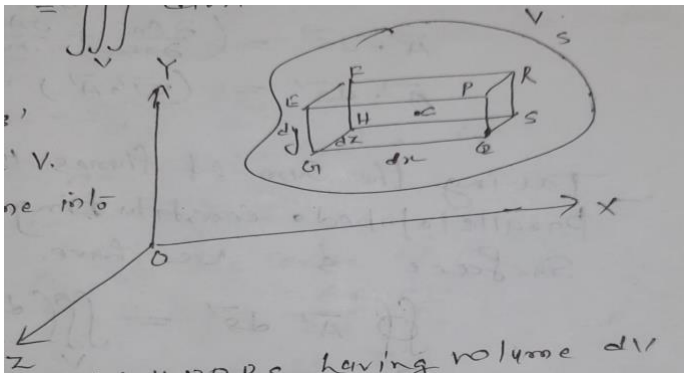
Consider a surface $'s'$ which encloses a volume V . Let us divide this volume into a large no of elementary volumes in the form of parallelepiped. Consider one such parallelepiped EFGHPQRS having volume dV and sides $dy\, dz$. consider a vector $\vec{A}$ at the centre 'c' of a parallelepiped. Let A_x, A_y & A_z = components of $\vec{A}$ at c along three axes.

The value of $x-$ component of $\vec{A}$ at the centre of face EFGH = $A_x - \frac{\partial A_x}{\partial x}.\frac{dx}{2}$

And that at the centre of face PQRS = $A_x + \frac{\partial A_x}{\partial x}.\frac{dx}{2}$

Since the volume element is infinitesimally small, this component of vector may be considered all over the face. Flux entering the face EFGH = $\left(A_x - \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right) dy\, dz$

Similarly flux leaving the face PQRS = $\left(A_x + \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right) dy\, dz$

Thus net flux leaving the parallelepiped in the $x-$ direction

$= \left[\left(A_x + \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right) - \left(A_x - \frac{\partial A_x}{\partial x}.\frac{dx}{2}\right)\right] dy\, dz = \frac{\partial A_x}{\partial x} dx\, dy\, dz$

Similarly, the net flux leaving the parallelepiped in the y & $z-$ directions are

$\frac{\partial A_y}{\partial y} dx\, dy\, dz$ and $\frac{\partial A_z}{\partial z} dx\, dy\, dz$

Total flux of $\vec{A}$ leaving from the parallelepiped

$\vec{A}.\vec{ds} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) dx\, dy\, dz = (\vec{\nabla}.\vec{A})dV$ where $div\, \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

and $dV = dx\, dy\, dz$

Taking the sum of fluxes through all the elementary parallelepipeds constituting the volume V of the surface S we have

$$\oint_S \vec{A}.d\vec{S} = \iiint_V (\vec{\nabla}.\vec{A})\,dV$$

<div align="right">Proved</div>

5. (a) The displacement of a simple harmonic oscillator at any instant of time t is given by
$$y = a\sin(\omega t + \varphi)$$
The velocity is defined as the time rate of change of displacement.

So Velocity $v = \dfrac{dy}{dt} = \dot{y} = a\omega\cos(\omega t + \varphi)$.

$v = a\omega\cos(\omega t + \varphi)$.

$= a\omega\sin\left(\omega t + \varphi + \dfrac{\pi}{2}\right)$,   as $\cos\theta = \sin\left(\dfrac{\pi}{2} + \theta\right)$

Since,

$\sin(\omega t + \varphi) = \dfrac{y}{a}$, $\cos(\omega t + \varphi) = \sqrt{1 - \dfrac{y^2}{a^2}} = \dfrac{\sqrt{a^2-y^2}}{a}$

$v = \omega\sqrt{a^2 - y^2}$

(b) The displacement of a simple harmonic oscillator at any instant of time t is given by
$$y = a\sin(\omega t + \varphi)$$
The velocity of a simple harmonic oscillator at any instant of time t is given by

$v = a\omega\cos(\omega t + \varphi) = a\omega\sin\left(\omega t + \varphi + \dfrac{\pi}{2}\right)$.

Comparing these two equations, we find that the velocity of simple harmonic oscillator at
Any instant of time t leads the displacement by a phase difference $\dfrac{\pi}{2}$ radian or 90^0

i.e. the two are in quadrature. The velocity varies harmonically with the same frequency ω.

And $v = \omega\sqrt{a^2 - y^2}$

i.e. the velocity will be minimum when displacement will be maximum.
<div align="center">i.e. $v = 0$ when $y = a$</div>
and velocity will be maximum when displacement will be zero.
$$v_{max} = a\omega$$
So both displacement and velocity are opposite to each other that introduced a phase difference
of $\dfrac{\pi}{2}$

6. The angular deformation of an object caused by a few twisting torques is known as the angle of twist.

consider a short cylinder of length l and radius a clamped at the upper end AB. Suppose a twisting couple is applied to the face A'B' as shown by the arrow head, in a direction perpendicular to the length of the cylinder. The radius O'P is twisted through an angle θ to the position O'P'.
θ is known as the angle of twist.

     Let a solid cylinder consists of a large number of Co-axial cylindrical shells. Let us consider one such cylindrical Shell of radius x and thickness dx
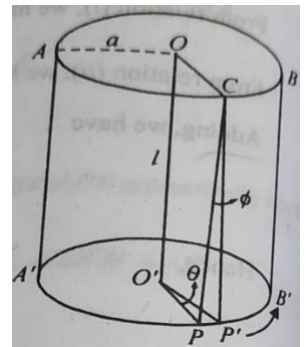
The angle of shear φ will have the maximum value when $x = a$ and least at O' where $x = 0$

This shows that shearing strain is not constant throughout the cylinder. It is maximum on the rim and least for the innermost layer.

If the points Q and Q' are supposed to lie on the rim of the hollow cylinder
Of radius x and φ is the angle of shear, then we have QQ'= $l\varphi$ (as φ is small)

QQ'=$x\theta$ i.e. $l\varphi = x\theta$ or $\varphi = \frac{x\theta}{l}$

If η is the coefficient of rigidity, then $\eta = \frac{Shearing\ stress}{Angle\ of\ shear} = \frac{T}{\varphi}$

$T = \eta\varphi = \frac{\eta x\theta}{l}$

Now, the face area of the hollow cylinder=$2\pi x.\,dx$

Total shearing force on this area $= 2\pi x dx.\frac{\eta x\theta}{l} = \frac{2\pi\eta\theta}{l}x^2 dx$

Moment of the force about OO'$= \frac{2\pi\eta\theta}{l}x^2 dx.\,x$

The twisting couple applied to the whole cylinder can be obtained by integrating this quantity for Limits $= 0\ and\ x = a$.

Hence twisting couple $= \frac{2\pi\eta\theta}{l}\int_0^a x^3 dx = \frac{2\pi\eta\theta}{l}\left[\frac{a^4}{4}\right] = \frac{\pi\eta\theta a^4}{2l}$

So this is the expression for the couple required to twist a uniform solid cylinder by an angle.

7. The coefficient of viscosity is thus defined as the tangential force per unit area required to maintain a unit velocity gradient, i.e. a unit relative velocity between two layers a unit distance apart.
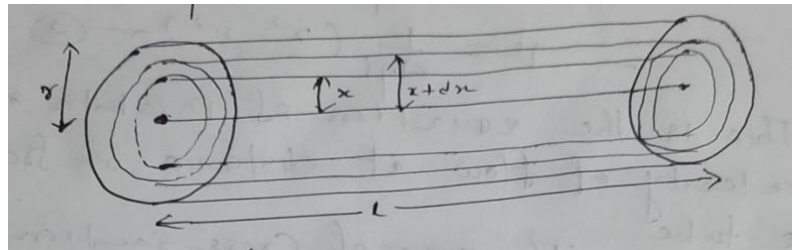
Consider a capillary tube of length l and radius . Let a liquid be maintained to flow through it.

It is assumed that

(i)   The tube is horizontal and so acceleration due to gravity is neglected.

(ii)   The motion is stream line and all the stream line flows are parallel to the axis of tube.

(iii)   The velocity of liquid along the wall is zero and is maximum along the axis of the tube.

A cylindrical layer of the liquid of radius x and thickness dx is considered.

The viscous force acting on the layer in back ward direction is given by

$F = -\eta A\frac{dv}{dx}$ where $\frac{dv}{dx}$ = velocity gradient.

The forward push due to the difference of pressure P on the two sides of the cylinder of radius x is given by $= P\pi x^2$. for steady flow

$-\eta A\frac{dv}{dx} = P\pi x^2$,   but $A = 2\pi xl$, $P\pi x^2 = -\eta 2\pi xl\frac{dv}{dx}$

Or $= -\frac{P}{2\eta l}x dx$, integrating we get $v = -\frac{P}{2\eta l}\frac{x^2}{2} + c$,   where c= integration constant

When $x = r, v = 0$, $c = \frac{P}{4\eta l}r^2$

So $v = \frac{P}{4\eta l}(r^2 - x^2)$

This is the equation of parabola and gives the velocity of flow at distance x from the axis of the tube.

The area of cross-section of the cylindrical layer of radius x and thickness dx is given by
$= 2\pi x dx$

Volume of liquid flowing per second through this area $dV = v2\pi x dx$

Hence volume of the liquid flowing out per second through whole tube is given by

$$\int dV = \int_0^r v2\pi x dx$$

$$V = \int_0^r \frac{P}{4\eta l}(r^2 - x^2)\, 2\pi x dx$$

$$= \frac{\pi P}{2\eta l}\int_0^r (r^2 x - x^3)dx$$

$$= \frac{\pi P}{2\eta l}\left(\frac{r^4}{2} - \frac{r^4}{4}\right)$$

$$V = \frac{\pi P r^4}{8\eta l}$$

This is Poiseuille's formula.

8. The Lorentz transformation equations are

(i) $\quad x' = \dfrac{x - vt}{\sqrt{\left[1 - \left(v^2/c^2\right)\right]}}$

(ii) $\quad y' = y$

(iii) $\quad z' = z$ and

(iv) $\quad t' = \dfrac{t - \left(vx/c^2\right)}{\sqrt{\left[1 - \left(v^2/c^2\right)\right]}}$

LEHGTH CONTRACTION:-

Let us consider two coordinate systems S and S', the latter moving with velocity v
Relative to former along (+)ve direction of
$x -$ axis. Let a rod of proper length l_0
(length at rest) be at rest in frame S' along
The $x -$ axis . if x_1' and x_2' are the coordinates
of abscissae of the ends of the rod, in S' at the
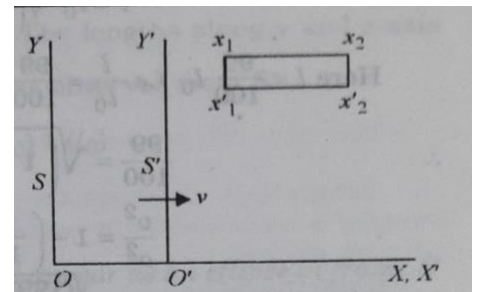same time t', then $l_0 = x_2' - x_1'$, since rod is at rest in frame S' .

If l is the length of the rod in frame S and the coordinates of abscissae of the ends of the rod in this system are x_1 and x_2 , at the same time t , then $l = x_2 - x_1$

According to Lorentz transformations,

$x_2' = \dfrac{x_2 - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$, $x_1' = \dfrac{x_1 - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$,   subtracting these equations we have

$x_2' - x_1' = \dfrac{x_2 - x_1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

Or $l_0 = \dfrac{l}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ or $l = l_0\sqrt{\left(1-\dfrac{v^2}{c^2}\right)}$

From this equation we see that $l < l_0$

Thus the length of the rod is reduced in the ratio $\sqrt{\left(1-\dfrac{v^2}{c^2}\right)} : 1$ as measured by moving observer with velocity v relative to rod along its length.

TIME DILATION:-

Consider two systems S and S', S' is moving with velocity v relative to S along positive direction of $x-axis$.

Let the clock be situated at x in frame S and gives signals at intervals.

$$\Delta t = t_2 - t_1$$

Then the interval observed by observer in system S' will be

$$\Delta t' = t_2' - t_1'$$

From Lorentz transformations we have

$$t_1' = \dfrac{t_1 - \left(\frac{vx}{c^2}\right)}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}, \qquad t_2' = \dfrac{t_2 - \left(\frac{vx}{c^2}\right)}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

Subtracting these equations we get

$$t_2' - t_1' = \dfrac{t_2 - t_1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$\Delta t' = \dfrac{\Delta t}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \quad \text{or} \quad \Delta t < \Delta t'$$

Thus the time interval Δt appears to the moving observer to the dilated or lengthened because the time interval in system S' greater than that in system S. We may state " A clock will be found to run more and more slowly if the relative velocity between the clock and observer is increased more and more".

9. According to the kinetic theory the molecules of a gas are moving constantly in all directions with various speeds. If in a gas the mass-velocity, temperature or molecular concentration varies from one part of the gas to another, then the moving molecules transport respectively the momentum, kinetic energy or mass from one part to another in order to bring equalization throughout the gas. This gives rise to viscosity, thermal conduction or diffusion respectively. Thus viscosity, conduction and diffusion represent respectively the transport of momentum, energy and mass. Hence these phenomena are called transport phenomena.

Let us consider a gas flowing in parallel layers between two horizontal planes PQ and LM. The velocity of flow of the Gas above an intermediate plane AB is Greater than that of the gas below AB. The layer above AB exerts a tangential Force on the layer below AB tending to
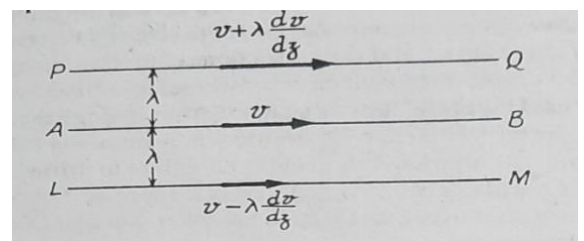
Accelerate it forward, while the layer Below AB exerts an equal force on the layer above AB tending to retard it backward. Thus these forces tend to reduce the difference in velocities on the two sides of AB. This property of the gas is called its viscosity.

The molecules above AB have a greater velocity of flow and hence possess a greater average momentum in the direction of flow than those below it. Now, on account of thermal agitation, there is a transfer of molecules of greater average momentum downwards across AB, and of molecules of smaller average momentum upwards across AB. Thus the layer of the gas above AB suffers a loss in momentum, whereas the layer below AB suffers a gain in momentum. By Newton's law of motion, a loss in momentum gives rise to a retarding force, and a gain in momentum gives rise to an accelerating force. Therefore, the layer above AB experiences a retarding force in the backward direction, while the layer below AB experiences an accelerating force in the forward direction. Thus viscosity is accounted for.

Let n be the number of molecules per unit volume of the gas and $\bar{c}$ their average velocity due to thermal agitation with which they are constantly moving in all directions. Let us consider the molecules crossing unit area of the plane AB from two planes PQ and LM situated at a distance λ (mean free path of the molecules) from AB. On arriving at AB they will make their first collisions there, and hence their first transfer of momentum.

Let v be the velocity of flow of the gas along the plane AB, and dv/dz the velocity gradient measured in a direction from PQ to LM perpendicular to AB. Then the velocity of the gas along the planes PQ and LM will be $\left(v + \lambda \frac{dv}{dz}\right) and \left(v - \lambda \frac{dv}{dz}\right)$ respectively. Now, the number of molecules crossing unit area of AB downwards in one second is $\frac{n\bar{c}}{6}$, and an equal number crossing upwards. Thus, if m be the mass of a molecule, the total mass crossing unit area of AB per second either way is $\frac{1}{6}mn\bar{c}$. The momentum carried downwards by the molecules crossing unit area of AB per second from the plane PQ is , therefore ,

$$= mass \times velocity$$
$$= \frac{1}{6}mn\bar{c} \times \left(v + \lambda \frac{dv}{dz}\right),$$

And similarly the momentum carried upwards

$$= \frac{1}{6}mn\bar{c} \times \left(v - \lambda \frac{dv}{dz}\right),$$

Hence the net momentum transferred per second downwards

$$= \frac{1}{6}mn\bar{c} \times \left(v + \lambda \frac{dv}{dz}\right) - \frac{1}{6}mn\bar{c} \times \left(v - \lambda \frac{dv}{dz}\right),$$
$$= \frac{1}{3}mn\bar{c}\lambda \frac{dv}{dz}.$$

Since this expression is the rate of change of momentum, it gives the tangential accelerating force which the upper layer exerts on AB.

Now, the coefficient of viscosity η is defined as the ratio of the tangential stress to the velocity gradient.

$$\eta = \frac{\frac{1}{3}mn\bar{c}\lambda \frac{dv}{dz}}{\frac{dv}{dz}}$$

$$\eta = \frac{1}{3} mn\bar{c}\lambda$$

But the mean free path $\lambda = \frac{1}{\sqrt{2}\pi nd^2}$, where d is molecular diameter.

$$\eta = \frac{1}{3\sqrt{2}} \frac{m\bar{c}}{\pi d^2}$$

This expression is free of . hence we conclude that for a given temperature the coefficient of viscosity of a gas is independent of pressure.

The above expressions show that the coefficient of viscosity of a gas is directly proportional to the average molecular velocity $\bar{c}$.

Where $\bar{c}$ is given by $\bar{c} = \sqrt{\frac{8 k T}{\pi m}}$

Hence the coefficient of viscosity of a gas varies directly as the square-root of the absolute temperature of the gas.

10. (i) A cycle of process:- when a closed system is taken from an initial to a final state by one or more processes, and then back to the initial state by some other one or more processes, the net change in internal energy of the system is zero ($\Delta U = 0$). This is because the internal energy of the system depends only on the state. Hence by the first law of thermodynamics, $\Delta U = Q - W$, we have

$0 = Q - W$

Or $Q = W$

In the differential form we may write $\oint dQ = \oint dW$

Thus, for a closed system undergoing a cycle of processes, the cyclic integral of heat is equal to the cyclic integral of work. This is also taken as a statement of the first law of thermodynamics.

(ii) Isobaric Process:- A process taking place at constant pressure is called an isobaric process. The work done by the mass in expanding from volume V_i to volume V_v against the constant external pressure p is $W = \int_{V_i}^{V_v} p \, dV = p \int_{V_i}^{V_v} dV = p(V_v - V_i)$

If ΔU be the change in the internal energy, then from the first law of thermodynamics, we have $\Delta U = Q - W = mL - p(V_v - V_i)$

this is the expression for the change in the internal energy of the system.

(iii) Isochoric process:- A process taking place at constant volume $(\Delta V = 0)$ is called an isochoric process. In such a process, the work done on or by the system is zero $(W = 0)$. Hence by the first law of thermodynamics, $\Delta U = Q - W$, we have $\Delta U = Q$,

Thus in an isochoric process the heat added to (or taken from) the system becomes Entirely the increase (or decrease) in the internal energy of the system.

(iv) Adiabatic process:- when a system passes from an initial state i to a final state f through a process such that no heat flows into or out of the system, then the process is called an adiabatic process. Such a process can occur when the system is perfectly insulated from the surroundings, or when the process is very rapid so that there is little time for the heat to flow into or out of the system.

Let a system pass from an initial state i to a final state f by absorbing heat Q from outside and by doing external work done W. If U_i and U_f be the internal energies of the system in the initial and final states respectively then, from the first law of thermodynamics, the change in energy is given by

$$\Delta U = U_f - U_i = Q - W$$

If the process be adiabatic then $= 0$. then the change in internal energy is $\Delta U = -W$ which is negative. Thus if an amount of work is done by the system in an Adiabatic process, the internal energy of the system decreases by that amount. That is the system does adiabatic work at the cost of its own internal energy. If, on the other hand the work is done on the system in the adiabatic process, the energy of the system increases by that amount.

(v)   Free Expansion:- if a system, say a gas, expands in such a way that no heat enters or leaves the system and also no work is done by or on the system, then the expansion is called the free expansion.

i.e $Q = 0\ and\ W = 0,$ so $U_f - U_i = 0,$ or $U_f = U_i$

thus the initial and the final internal energies are equal in free expansion.